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ON THE INTERACTION OF SHOCK WAVES IN WATER-SATURATED EARTH AND IN WATER

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ON THE INTERACTION OF SHOCK WAVES IN WATER-SATURATED EARTH AND IN WATER

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(Moscow)

On the basis of an approximation of the dependence p(V) by a piecewise linear function we give an approximate solution to the problem of the interaction of a two-dimensional nonstationary shock wave with the boundary of media, a shifted boundary, and with a boundary of a region of given pressure.

1. The interaction of a shock wave with a boundary of media or a shifted boundary. In water, petroleum, water with air bubbles, water-saturated ground, and certain other media during shock compression, as is known [1-4], the pressure may be viewed only as a function of the specific volume V

$$p = p(V) \qquad (1.1)$$

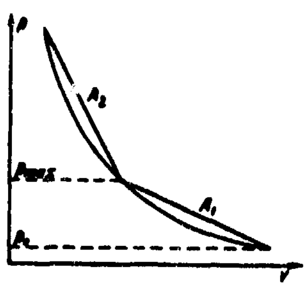

Fig. 1.

The relationship p(V) is nonlinear, but it is identical with a rise or fall in pressure. Thus $p' < 0$, and $p'' > 0$. In such media, called nonlinearly elastic, the basic equations of gas dynamics, expressing the laws of conservation of mass and quantity of motion, and also the relationships on the shock wave front, conforming to these same laws, together with (1.1) form a closed system and allow us to view the propagation and interaction of shock waves without using a thermodynamic relationship. A thermodynamic relationship remains in force. Using it, we may determine the entropy losses on the wave front.

The propagation of a stationary wave an a medium with variable sign p'' was examined by G. I. Barenblatt [5] on the basis of [6]. The propagation of a nonstationary wave was examined in [7].

In the segment h = 0 when t = 0, let the pressure from the shock increase from $p_0 = 0$ to p_{max} and fall according to a certain law

$$p = f(t) \tag{1.2}$$

At distance h^* from the initial segment the medium borders on the second medium. On the border there may be a barrier made of incompressible material, the mass of which is equal to m for a unit of area.

In a system of LaGrange coordinates (mass h, time t) a solution of the equations of gas dynamics (cf., for example, [7])

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \qquad \frac{\partial u}{\partial h} - \frac{\partial V}{\partial t} = 0$$

in the case of a linear dependence between the volume and the pressure has the form

$$\begin{gathered} p = F_1(h - At) + F_2(h + At) \\ (A = \rho c, \quad \rho = 1/V) \\ u = \frac{1}{A}[F_1(h - At) - F_2(h + At)] \end{gathered} \tag{1.3}$$

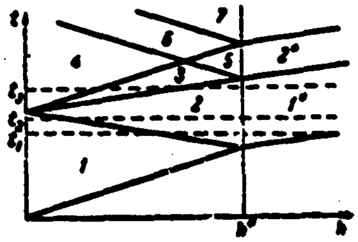

Fig. 2.

Here and further u is the velocity of particles, A is the acoustic resistance of the medium, and c is the velocity of sound. Functions F_1 and F_2 are determined by the initial and boundary conditions.

Let us approximate the curve p = p(V) in the first medium by the two straight lines (Fig. 1):

$$\begin{aligned} p &= -A_1 V + B_1 \quad \text{when} \quad 0 < p < p_{max} \\ p &= -A_2 V + B_2 \quad \text{when} \quad p > p_{max} \end{aligned} \qquad (1.4)$$

The number of approximating links may be increased depending on the required accuracy of the calculation. From segment h = 0 the front of the shock wave will spread (Fig. 2). The solution in area 1 (elastic wave) we will obtain in the form

$$p = f\left(\frac{-h + A_1 t}{A_1}\right), \qquad u = \frac{p}{A_1} \qquad (1.5)$$

The form of function f(t) here and henceforth is determined by equation (1.2).

At moment $t = h^*/A_1$ the front of the wave reaches the media boundaries. Let in the second medium the acoustic resistance A^* be constant and $A^* > A_2 > A_1$. In the reflection of the wave the pressure increases, and the acoustic resistance of the first medium becomes equal to A_2. When $t = h^*/A_1$ the velocity of the front of the reflected wave $h^{\cdot} = -A_2$.

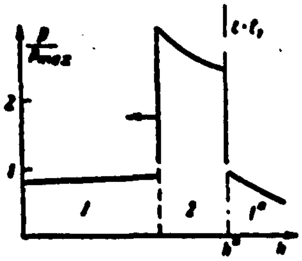

Fig. 3.

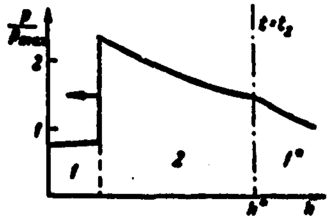

Fig. 4.

In dense media the velocity of the front changes little with the change in pressure. For example, in water with the fall in pressure on the front from 250 to 1 kg/cm^2 the velocity of the front D according to data [2] is reduced from 1530 to 1500 m/s. In a water-saturated ground [4], if before the wave front p_0 = = 25 kg/cm^2, a fall in pressure on the front from 75 to 50 kg/cm^2 is accompanied by a fall in the velocity of the front by 0.08 of the initial value of D with a content of entrapped air equal to 0.01 of the total volume of ground, and less than by 0.001 D in ground not containing air. Therefore, we may assume that $h^\circ = -A_2$ in the entire segment from $h = h^*$ to $h = 0$.

The solution in region 2 (see Fig. 1) is determined from the relationship on the front of the reflected wave

$$p_2 + A_2 u_2 = p_1 + A_2 u_1 = \frac{A_1 + A_2}{A_1} f\left(\frac{A_1 + A_2}{A_1} t - \frac{A_1 + A_2}{A_1^2} h^\circ\right)$$

and from the condition on the media boundary. The velocity of the medium particles touching the boundary on both sides is equal to the velocity of the boundary $\phi(t)$.

Taking into account the relationship on the front of the reflected wave and (1.3), we find the equation of motion of the boundary

$$m\dot{\varphi}(t) = \frac{A_1 + A_2}{A_1} f\left[\frac{A_1 + A_2}{2A_1^2}(A_1 t - h^\circ)\right] - (A_2 + A^\circ)\varphi(t) \qquad (1.6)$$

and the solution in region 2 in the form

$$
\begin{aligned}
p(h,t) &= \frac{A_1 + A_2}{2A_1} f\left[-\frac{A_1 + A_2}{2A_1 A_2}(h - A_2 t) + \frac{A_1^2 - A_2^2}{2A_1^2 A_2} h^*\right] + \\
&\quad + \frac{A_1 + A_2}{2A_1} f\left[\frac{A_1 + A_2}{2A_1 A_2}(h + A_2 t) - \frac{(A_1 + A_2)^2 h^*}{2A_1 A_2}\right] - A_2 \varphi\left(\frac{h - h^* + A_2 t}{A_2}\right) \\
u(h,t) &= \frac{A_1 + A_2}{2A_1 A_2} f\left[-\frac{A_1 + A_2}{2A_1 A_2}(h - A_2 t) + \frac{A_1^2 - A_2^2}{2A_1^2 A_2} h^*\right] - \\
&\quad - \frac{A_1 + A_2}{2A_1 A_2} f\left[\frac{A_1 + A_2}{2A_1 A_2}(h + A_2 t) - \frac{(A_1 + A_2)^2 h^*}{2A_1^2 A_2}\right] + \varphi\left(\frac{h - h^* + A_2 t}{A_2}\right)
\end{aligned}
\qquad (1.7)
$$

The solution in region 1* is

$$
p(h,t) = A^* \varphi\left(\frac{h^* - h + A^* t}{A^*}\right), \quad u(h,t) = \varphi\left(\frac{h^* - h + A^* t}{A^*}\right) \qquad (1.8)
$$

The form of function $\phi(t)$ is determined from equation (1.6).

If on the boundary of the media there is no barrier, the velocity of the boundary and the pressure on the boundary are determined by equation

$$
u = \dot{\varphi}(t) - \frac{A_1 + A_2}{(A^* + A_2) A_1} f\left[\frac{A_1 + A_2}{2A_1^2}(A_1 t - h^*)\right], \quad p = A^* \varphi(t) \qquad (1.9)
$$

The ratio of the pressure acting on the barrier to the pressure in the incident wave (the reflection coefficient) in a nonlinearly elastic medium is $\eta = (A_1 + A_2)/A_1 > 2$; and in a linearly elastic medium when $A_1 = A_2$ we obtain $\eta = 2$.

If the barrier is absent, then $\eta = \eta_1$ in a nonlinearly elastic medium and $\eta = \eta_2$ in a linearly elastic medium; here for each medium we will get

$$
\eta_1 = \frac{(A_1 + A_2) A^*}{(A^* + A_2) A_1} > 1, \quad \eta_2 = \frac{2A^*}{A_1 + A^*} > 1, \quad \eta_1 > \eta_2 ,
$$

respectively.

In the absence of a barrier the media boundary moves slowly, and $\dot{\phi}(t)$ decreases monotonically. In the reflected wave the

pressure has a minimum value on the front, and a maximum value when $h = h^*$. In the second medium maximum pressure is attained on the wave front, while minimum pressure occurs on the media boundary.

In the presence of a barrier of finite mass for a certain time after $t = t^*/A_1$ the barrier moves quickly, and $\phi(t)$ increases with the increase in the argument. The pressure has its greatest value on the front of the reflected wave, and its minimum value when $h = h^*$. This time interval is shown by Fig. 3. Figure 4 corresponds to the moment t_2, when $\phi(t)$ achieves a maximum. From here on the minimum value of pressure will be found at the point which approaches the wave front with the passage of time (Fig. 5). Moments t_1, t_2, and t_3, to which Figs. 3, 4, and 5 correspond, are represented in Fig. 2.

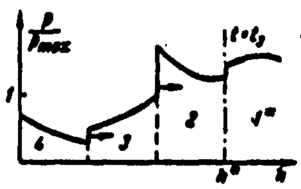

Fig. 5.

2. **The interaction of a shock wave with a boundary of an area of fixed pressure.** The solution in area 2 will be valid for the arrival of a front of a reflected wave to the segment $h = 0$.

Let us examine the case of large θ and small h^* and let us assume that in the reflected wave the acoustic resistance remains equal to A_2. In the plane $h = 0$ $p < p_{max}$. In the reflection of the wave from segment $h = 0$ a change in the acoustic resistance takes place and regions 3 and 4 are formed (see Fig. 2). In region 3 on the boundary with region 2 the pressure falls abruptly to p_{max}. In region 4 a further decrease in pressure occurs with the growth in time. On the boundary of regions 2 and 3

$$\Phi_1 + \Phi_2 = p_{max}, \qquad \Phi_1 - \Phi_2 = \frac{p_{max} - 2F_2}{A_2} A_3 \tag{2.1}$$

where F_2 corresponds to region 2, and Φ_1 and Φ_2 correspond to region 3.

From here in region 3

$$\begin{aligned}\Phi_1(h-A_1t)&=\frac{A_1+A_2}{2A_2}p_{\max}-\frac{A_1+A_2}{2A_2}f\left[\frac{(A_1+A_2)(h-A_1t)}{A_1(A_2-A_1)}+\frac{(A_1+A_2)^2h^\circ}{A_1A_2(A_2-A_1)}\right]+\\&+A_1\varphi\left[\frac{2(h-A_1t)}{A_2-A_1}+\frac{2A_1^2+A_1A_2+A_2^2}{A_1A_2(A_2-A_1)}h^\circ\right]\\\Phi_2(h+A_1t)&=\frac{A_2-A_1}{2A_2}p_{\max}+\frac{A_1+A_2}{2A_2}f\left[\frac{h+A_1t}{A_1}-\frac{A_1+A_2}{A_1A_2}h^\circ\right]-\\&-A_1\varphi\left[\frac{2(h+A_1t)}{A_1+A_2}-\frac{A_2-2A_1}{A_1A_2}h^\circ\right].\end{aligned}\tag{2.2}$$

The solution in region 3 is

$$p=\Phi_1+\Phi_2,\qquad u=\frac{1}{A_1}(\Phi_1-\Phi_2).$$

In segment $h = 0$ $p = f(t)$, and on the boundary of regions 3 and 4 the condition $p_4 - A_1u_4 = p_3 - A_1u_3 = 2\Phi_2$ is fulfilled. From here it follows that in region 4 the function Φ_2 is the same as in region 3, and

$$\begin{aligned}\Phi_1(h-A_1t)&=f\left(\frac{A_1t-h}{A_1}\right)-\frac{A_2-A_1}{2A_2}p_{\max}-\frac{A_1+A_2}{2A_2}f\left[\frac{A_1t-h}{A_1}-\frac{A_1+A_2}{A_1A_2}h^\circ\right]+\\&+A_1\varphi\left[\frac{2(A_1t-h)}{A_1+A_2}+\frac{A_2-2A_1}{A_1A_2}h^\circ\right].\end{aligned}\tag{2.3}$$

The solution in region 4 is

$$p=\Phi_1+\Phi_2,\qquad u=\frac{1}{A_1}(\Phi_1-\Phi_2)\tag{2.4}$$

The shock wave is reflected from the boundary of the area of fixed pressure in the form of a vacuum wave.

3. **The interaction of a vacuum wave with a media boundary.** At the moment when the vacuum wave front reaches the media boundary regions 5 and 2* are formed. Let Φ_1 and Φ_2 determine the flow in region 5, and let F_1 and F_2 determine the flow in region 2*. From the condition on boundaries 1*, 2*, and 3, 5 we obtain $F_2 = 0$, while Φ_1 is the same function as in region 3. In region 5 we have $p_5 + A_1u_5 = 2\Phi_1$, and in region 2* we have $p_2{}^* - A^*u_2{}^* = 0$.

The equation of motion of the barrier is

$$m\dot{g}(t) + (A_1 + A^*)g(t) - 2\Phi_1(h^* - A_1 t) = 0 \tag{3.1}$$

The pressure against the barrier from the side of region 5 is

$$p = 2\Phi_1(h^* - A_1 t) - A_1 g(t)\,, \tag{3.2}$$

the pressure from the side of region 2 is

$$p_2^* = A^* g(t) \tag{3.3}$$

If the barrier is fixed, and when h = 0 the pressure is fixed in the form

$$p = p_{max}\left(1 - \frac{t}{\theta}\right), \tag{3.4}$$

then the pressure against the barrier in region 5 is

$$p = \frac{(A_1 + A_2)^2 p_{max}}{A_1 A_2 (A_2 - A_1)\theta}\left(\frac{2A_2 + A_1}{A_2}h^* - A_1 t\right). \tag{3.5}$$

At the moment $t_m = h^*(2A_2 + A_1)/A_1A_2$ region 7 is formed. In this region the function Φ_1 is the same as in region 4. If in the segment h = 0 the pressure is fixed in the form (3.4), then when $h = h^*$ in region 7

$$\begin{aligned}\Phi_1(h^* - A_1 t) = -\frac{p_{max}}{2A_2^2\theta}[(A_2 - A_1)A_2 t - (A_1 + 3A_2)h^*] + \\ + A_1\varphi\left[\frac{2A_1 t}{A_1 + A_2} + \frac{A_2^2 - 2A_1^2 - 3A_1A_2}{A_1A_2(A_1 + A_2)}h^*\right].\end{aligned} \tag{3.6}$$

If the mass of the barrier is finite here, then when $t = t_m$ the velocity of the barrier may not change abruptly, and therefore it is determined by equation (3.3). By using (3.6) and taking into account that $\phi(h^*/A) = 0$, we will obtain the pressure against the barrier with $t = t_m$

$$p = 2\Phi_1(h^* - A_1 t_m) - A_1 g(t_m) = -\frac{2(A_1 + A_2)h^*}{A_1 A_2 \theta}p^*_{max} - A_1 g(t_m) < 0\,. \tag{3.7}$$

The pressure is negative. The medium attains a velocity directed from the barrier to the side of the segment h = 0, and breaks away from the barrier. Further on the pressure from the side of the second medium causes a reduction in the velocity of the barrier and the formation of contact with the first medium. The pressure arising here is less than in the examined regions.

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13. ABSTRACT

By using a piece-wise linear approximation for the volume as a function of the pressure, approximate solutions are given for three problems. The first problem is that of the interaction of a shock wave with the boundary of the medium or with a moving barrier. The propagation of a stationary wave in a region where the second derivative of the pressure with respect to the volume is variable in sign has been discussed by G. I. Barenblatt. Propagation of a non-stationary wave was considered in an earlier paper of the present author. At a given section there is a jump discontinuity in the pressure and at the section given the first medium is bounded by a second. On this boundary there can be a barrier of an incompressible material. Choosing h and the t as Lagrangian coordinates the equations of gas dynamics can be solved generally and the solution applied to various regions in which the problems of the paper arise. In the first proplem we are concerned with the propagation of a wave from the given section, its reflection from the section given and the propagation into the second medium, which depends on the velocity of the barrier between the two media. Expressions for the pressure and velocity are given in each case. The second problem is the interaction of a shock wave with the boundary of a domain of given pressure.

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